Chapter 1

INTERPOLATION

1. (a) Using `polint`, the interpolated value is 1.577.
   (b) See Fig. 1.1. Comparing to Example 1.1, the current interpolation is better around the center but much worse near the end points.

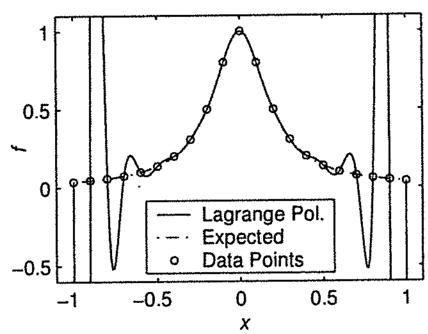

Figure 1.1: Exercise 1.

2. Differentiating $P(x) = \sum_{j=0}^{n} y_j \alpha_j \prod_{\substack{i=0 \\ i\neq j}}^{n} (x - x_i)$ gives

$$P'(x) = \sum_{j=0}^{n} y_j \alpha_j \frac{d}{dx} \prod_{\substack{i=0 \\ i\neq j}}^{n} (x - x_i) = \sum_{j=0}^{n} y_j \alpha_j \Big[\sum_{\substack{k=0 \\ k\neq j}}^{n} \prod_{\substack{i=0 \\ i\neq k,j}}^{n} (x - x_i) \Big].$$

3. When $g''(x_i) = g''(x_{i+1})$, the x^3 terms in (1.6) cancel out and $g_i(x)$ becomes a parabola:

$$\begin{aligned} g_i(x) &= \frac{g''(x_i)}{6}\Big[3x^2 - 3x(x_i + x_{i+1}) + 3x_i x_{i+1}\Big] + \\ & f(x_i)\frac{x_{i+1} - x}{\Delta_i} + f(x_{i+1})\frac{x - x_i}{\Delta_i}. \end{aligned}$$

4. (a) Continuity of the first derivative.

(b) For $x_i \le x \le x_{i+1}$:

$$g_i'(x) = g'(x_i)\frac{x - x_{i+1}}{x_i - x_{i+1}} + g'(x_{i+1})\frac{x - x_i}{x_{i+1} - x_i}.$$

Integrating and substituting $g_i(x_i) = f(x_i)$ and $g_i(x_{i+1}) = f(x_{i+1})$, we obtain

$$g'(x_i) + g'(x_{i+1}) = 2\frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}, \quad i = 0, \ldots, N-1$$

These are N equations for the $N+1$ unknowns $g'(x_0), \ldots, g'(x_N)$. One additional equation is required and it can be $g'(x_0) = g'(x_1)$, which means that the interpolant in the first interval is a straight line.

(c) For non-periodic equally-spaced data, the solution of (1.7) requires $O(2N)$ divisions and $O(3N)$ of each additions and multiplications, ignoring the effort in computing the right-hand side. Solving the system in (b) is only $O(N)$ additions.

5. Solve first for $g''(x_0), \ldots, g''(x_N)$ as explained in the text and then differentiate (1.6) to get the first derivative at the data points.

For $x_0 \le x_i \le x_{N-1}$:

$$g'(x_i) = g_i'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} - g''(x_i)\frac{h}{3} - g''(x_{i+1})\frac{h}{6}.$$

For x_N:

$$g'(x_N) = g_{N-1}'(x_N) = \frac{f(x_N) - f(x_{N-1})}{h} + g''(x_{N-1})\frac{h}{6} + g''(x_N)\frac{h}{3}.$$

6. (a) For $\sigma = 0$, (1.3) is recovered. For $\sigma \to \infty$ we obtain

$$g_i(x) = f(x_i)\frac{x - x_{i+1}}{x_i - x_{i+1}} + f(x_{i+1})\frac{x - x_i}{x_{i+1} - x_i},$$

which is a straight line.

(b) The given differential equation for g_i is second order, linear, and non-homogeneous. Its solution is:

$$\begin{aligned} g_i(x) \;=\; & C_1 e^{\sigma x} + C_2 e^{-\sigma x} - \frac{g''(x_i) - \sigma^2 f(x_i)}{\sigma^2}\frac{x - x_{i+1}}{x_i - x_{i+1}} \\ & - \frac{g''(x_{i+1}) - \sigma^2 f(x_{i+1})}{\sigma^2}\frac{x - x_i}{x_{i+1} - x_i}. \end{aligned}$$

Differentiating:

$$g_i'(x) = C_1\sigma e^{\sigma x} - C_2\sigma e^{-\sigma x} + \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i} - \frac{1}{\sigma^2}\frac{g''(x_{i+1}) - g''(x_i)}{x_{i+1} - x_i}.$$

C_1, C_2, and the second derivatives at the data points are determined as in Section 1.2 with (1.4) and (1.5) replaced by the two equations above.

7. (b,c) `polint`, `spline`, and `splint` are used to obtain the interpolations in Fig. 1.2. The predicted tuition in 2001 is \$10,836 using Lagrange polynomial and \$34,447 using cubic spline. The Lagrange polynomial does a pretty good job interpolating the data but behaves very poorly away from it; the predicted tuition is way too low. The cubic spline behaves well for both interpolation and extrapolation.

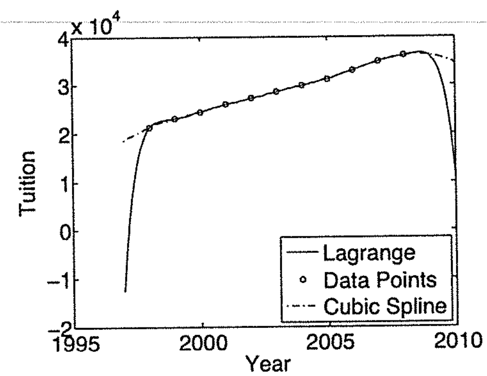

Figure 1.2: Exercise 7.

8. (a) Using `polint`, the interpolation is shown in Fig 1.3. The prediction in 2009 is -38.40 which is unrealistic.

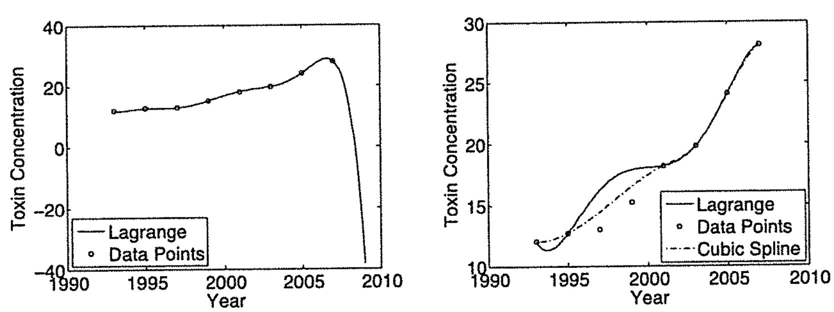

Figure 1.3: Exercise 8.

(b,c) Results are shown in Fig. 1.3. The predicted values are

	Lagrange	Spline
1997	16.23	14.44
1999	17.88	16.52

The predictions using the cubic spline are better.